High Data Volume Seismology: Surviving the Avalanche

by

Henry Philip Crotwell

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University of South Carolina, 1992

Master of Arts
University of South Carolina, 1994

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Major Professor

Committee Member

Committee Member

Chairman, Examining Committee

Dean of The Graduate School
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ABSTRACT

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Seismic data volumes have increased in the past twenty years with the Incorporated Research Institutes for Seismology’s Data Management Center currently archiving upwards of 14 terabytes per year and this trend will continue (Ahern, 2006). Data volumes are quickly reaching the point at which the individual seismologist can be overwhelmed with the avalanche of data. We present three studies at the intersection of seismology and software development that aim to enable more efficient use of data by practicing seismologists. The first is the TauP Toolkit which calculates the travel times of seismic waves through custom one dimensional earth models. TauP also allows almost arbitrary phases to be used and is incorporated into a wide variety of seismology software. TauP is available at http://www.seis.sc.edu/TauP. The second is a study of the compression of seismic data, allowing more efficient storage and transmission. We find that the predictive operator used can have a significant effect on the compression used, and in many cases second differencing can be noticeably better than first differencing. The last is the EarthScope Automated Receiver Survey, which aims to calculate bulk crustal properties for all three component broadband seismic stations available in the US in a highly automated manner. Because of the high degree of automation, the project has been extended to calculated crustal thickness and \( V_p/V_s \) for global stations as well. Results are available at http://www.seis.sc.edu/ears.

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Chapter 1

Introduction

The use of computers within seismology has become central within the last few decades, and the change has enabled huge increases in our ability to learn about the earth. Many techniques that were simply impossible with paper records have become commonplace, and led to the increasing reliance of software specific to seismology within the research community. This transition from analog to digital recording has also fueled a tremendous increase in the volume of seismic data available to researchers over the last two decades. Foremost perhaps is the centralized storage and dissemination provided by the Incorporated Research Institutes for Seismology’s Data Management Center, allowing much easier access to existing data. Enhancements to recording systems, following the general increase in computer storage, has allowed the migration from event-windowed to continuously recorded data, as well as increases in sampling rates. The IRIS DMC has ingested many new networks over the years, including regional US networks, foreign national networks and temporary networks, making these data sets much more readily available. In addition, these individual networks have increased the number of stations within them. All of these factors have had a multiplicative effect on data volumes, leading to the change from a comparative trickle at the beginning of IRIS and the digital recording age to the veritable flood today shown in Figure 1.1.
Figure 1.1: Growth of the archive of seismic data at the IRIS DMC over the past 25 years.

While no one ever wishes for less data, many seismic analysis and processing techniques of the past can be overwhelmed by this flood if they continue to be used in the same way. The data volumes have grown by several orders of magnitude, while the way that seismologists process data has not undergone a similar revolution. Although the toolkit available to researchers has improved, in many ways the same scripts are being run, the same manual searches, the same data retrievals and the same manual processing steps are still employed much as they were a decade ago.

Perhaps because of the limitations of the tools available, most analysis tends to be done by applying a technique to a given geographical region. While there are many good reasons to focus on a single locale, in some sense the only natural data set for seismology is the entire Earth. Except for the constraints of time, there is
little reason not to apply a processing system to the entire globe. Of course time is always in short supply, at least that of the seismologist. However, if a technique can be automated so that it can continue to process data independent of a person, then the focus can be expanded to include all available data since computer time is rarely in short supply. This highly automated processing notion can not only cope with the increasing volumes of seismic data, but thrives on it, allowing greater use of stacking and statistics to supplant the “by hand” intensive study of more traditional seismic processing. Currently there are about 700 stations at the IRIS DMC available in near real time, shown in Figure 1.2, and only an automated system can hope to deal with this kind of continual new data. We present three studies at the interface of seismology and software development: 1D global seismic travel time calculations, compression of seismic data and automated bulk crustal properties from receiver functions.

![Figure 1.2: The 700 stations currently available in near real time at the IRIS DMC.](image-url)

The first is the TauP Toolkit, an implementation of a global 1D seismic wave travel time calculator. This brings the ability to calculate travel times within a wide
variety of custom earth models as well as for a nearly all seismic phases. The TauP Toolkit is very flexible and can be embedded within existing applications. Within the past year there have been almost 500 downloads of the latest version. This chapter was previously published as Crotwell et al. (1999).

The second is a study of the compression of seismic data, allowing more efficient storage and transmission of waveforms. The predictive operator used can have a large impact on the compression ratio, and we find that second differencing can be a significant improvement over first differencing in many cases.

The last is EARS, the EarthScope Automated Receiver Survey, which demonstrates a highly automated seismic processing system, capable of handling huge volumes of waveforms. EARS uses receiver functions to calculate bulk crustal properties, thickness and $V_p/V_s$, for all broadband three-component seismic stations available from the Incorporated Research Institutes for Seismology’s Data Management Center (IRIS DMC). EARS uses the $H\kappa$ stacking technique of Zhu and Kanamori (2000) and to date contains over 180,000 receiver functions for over 1700 stations around the globe.
Chapter 2

The TauP Toolkit

2.1 Introduction

The calculation of travel-times and ray paths of seismic phases for a specified velocity model of the earth has always been a fundamental need of seismologists. In recent years, the number of different phases used in analysis has been growing as has the availability of new earth models, especially models developed from detailed regional studies. These factors highlight the need for versatile utilities that allow the calculation of travel-times and raypaths of (ideally) any conceivable seismic phase passing through (ideally) arbitrary velocity models. The method of Buland and Chapman (1983) provides significant progress toward this need by allowing for the computation of times and paths of any rays passing through arbitrary spherically-symmetric velocity models. The implementation of this method through the ttimes software program (Kennett and Engdahl, 1991) for a limited number of velocity models and a standard set of seismic phases has been widely used in the seismological community. In this paper, we describe a new implementation of the method of Buland and Chapman (1983) that easily allows for the use of arbitrary spherically-symmetric velocity models and arbitrary phases. This package, the TauP Toolkit, provides for the computation of phase travel-times, travel-time curves, raypaths through the earth,
and phase piercing-points in a publicly-available, machine-independent package that should be of use to both practicing seismologists and in teaching environments.

### 2.2 Methodology

The method of *Buland and Chapman* (1983) works in the delay (or intercept) time ($\tau$) - ray parameter ($p$) domain to avoid complications of multi-valued travel-time branches associated with working in the time-distance domain directly. Physically, $\tau$ is the zero-distance intercept and $p$ is the slope of the line tangent to the travel-time curve at a given distance. The advantages of working in this domain are that $\tau$-branches are monotonic and single valued in all cases, thus do not suffer from the problems of triplications as their corresponding travel-time branches may in some circumstances. Furthermore, the ray parameters that define each $\tau$ branch are simple and well-defined functions of the earth model. We will not describe the method in any detail, but we will discuss the general steps necessary to implement this approach.

We used Maple (*Heal et al.*, 1996), a symbolic mathematics utility, to help convert the equations in *Buland and Chapman* (1983) to algorithmic forms for a spherical coordinate system, avoiding the additional complication of an earth flattening transform. The resulting Maple files are provided with the TauP Toolkit distribution for those interested in a more detailed study of the methodology.

Generating travel-times using the method of *Buland and Chapman* (1983) consists of four major steps. First, sampling the velocity-depth model in slowness. Next, integrating the slowness to get distance and time increments for individual model branches. A “branch” of a model is a depth range bounded above and below by local slowness extrema (first-order discontinuities or reversals in slowness gradients). A sum of these model branches along a particular path results in the corresponding branch of the travel time curves. These first two steps are undertaken in the *taup_create* utility.
and need only be done once for each new velocity model. The third step involves summing the branches along the path of a specified phase. Finally, an interpolation between time-distance samples is required to obtain the time of the exact distance of interest. These final steps are undertaken in various TauP Toolkit utilities, depending on the information that is desired by the user.

2.2.1 Slowness Sampling

Creating a sufficiently dense sampling in slowness from the velocity models is the most complicated and crucial step. There are several qualifications for a sufficiently dense sampling. First, all critical points must be sampled exactly. These include each side of first-order discontinuities and all reversals in slowness gradient. Less obvious points that must be sampled exactly occur at the bottom of a low velocity zone. This sample is the turning point for the first ray to turn below the zone. Note, it is possible for the velocity to decrease with depth but, due to the curvature of the earth, there may not be any of the pathological effects. For example, PREM (Dziewonski and Anderson, 1981) contains a low velocity layer below the moho, but rays turn throughout it. Hence, a more strictly accurate term might be “high slowness zones”.

A second sampling condition is that slowness samples must be sufficiently closely spaced in depth. This is normally satisfied by the depth sampling interval of the velocity model itself, but additional samples may need to be inserted. A third condition is that the sampling interval must be sufficiently small in slowness. We satisfy this condition by inserting slowness samples whenever the increment in slowness is larger than a given tolerance, solving for the corresponding depth using the original velocity model.

Finally, the resulting slowness sampling must not be too coarse in distance, as measured by the resulting distance sampling for the direct P or S wave from a surface source. Our approach to satisfying this condition is similar to that of the previous
condition. We insert new slowness samples whenever the difference in total distance
between adjacent direct rays from a surface source exceeds the given tolerance. Again,
the depth corresponding to the inserted slowness sample is computed from the original
velocity model. As an additional precaution against undersampling, we also check to
make sure that the curvature is not too great for linear interpolation to be reasonable.
This is accomplished by comparing the time at each sample point with the value
predicted from linear interpolation from the previous to the next point. If this exceeds
a given tolerance then additional samples are inserted before and after the point. The
last two conditions require the most computational effort, but also have the most effect
in creating a sufficiently well-sampled model. The sampling for P and S must meet
these conditions individually and, in order to allow for phase conversions, must be
compatible with each other.

2.2.2 Branch Integration, Phase Summing and Interpolation

For each region, or branch, of the model, we merely sum the distance and time contri-
butions from each slowness layer, between slowness samples, for each ray parameter.
Within each layer we use the Bullen law $v(r) = Ar^B$, which has an analytic solution.
The ray parameters used are the subset of slowness samples that correspond to turn-
ing or critically reflected rays from a surface source. Care must be taken to avoid
summing below the turning point for each slowness.

Once these “branches” have been constructed, it is straightforward to create a
real branch of the travel time curves by an appropriate summing. Of course, the
maximum and minimum allowed ray parameter must be determined as well in order
to assure that the ray can actually propagate throughout these regions. For instance,
the branch summing for a P wave turning in the mantle and for PcP are the same.
However, the ray parameter for any P phase must be larger than the slowness at the
bottom of the mantle, while that for PcP must be smaller.
Once the sum has been completed to create time, distance and tau as a function of ray parameter, an interpolation between known points must be done to calculate arrivals at intermediary distances. Currently, we use a simple linear interpolate which is sufficient for most purposes. More advanced interpolates do provide advantages in reducing the number of samples needed to achieve a given accuracy, but they can have certain instability or oscillatory problems that are difficult to deal with when the model is not known in advance.

Figure 2.1 and 2.2 summarize the effect of two choices of sampling parameters using the IASP91 model (Kennett and Engdahl, 1991). In Figure 2.1, we illustrate that our normal sampling (model file: iasp91) produces a relatively small residual, maximum of 0.013 seconds for P and 0.016 seconds for S, relative to a highly oversampled version of the same model generated to produce the most accurate time estimates without regard to model size or computation time. Thus, model iasp91 is likely the appropriate choice for travel times that will be used in further computations, for instance earthquake location studies. However, the model size is 337Kb which may be slow to load on some computer systems and does increase the computation time somewhat. A more coarsely sampled version of IASP91, which we distribute in model file qdt (quick and dirty times), sacrifices some accuracy for a smaller model file. Figure 2.2 shows qdt residuals with respect to the same highly oversampled model. There are larger and more noticeable peaks due to linear interpolation between the more widely spaced samples. While this would not make a good choice for earthquake location work, an error of 0.25 seconds is entirely satisfactory for classroom work, for determining windows for extracting phases, and for getting quick estimates of arrival times. Most important is the fact that the decision of how accurately the model needs to be sampled is left up to the user. They can easily create new samplings of existing models tailored to the requirements of the job at hand.

Figure 2.3 compares TauP results using IASP91 to output from ttimes. It would
Figure 2.1: First arriving P (top) and S (bottom) residual between a highly oversampled IASP91 model and the default sampling. The time residual is sampled every tenth of a degree and is within 0.013 seconds for P and 0.016 seconds for S over the entire distance range. The slight trend to increasingly negative residuals with distance is the result of using a less dense linear interpolation of the original cubic IASP91 velocity model. While the difference associated with this trend is small, about 0.01 seconds at 180 degrees, adding the ability to read cubic splines directly should eliminate it.
Figure 2.2: First arriving P (top) and S (bottom) residual between a highly oversampled IASP91 model and a coarsely sampled model. This less well sampled version of IASP91 is referred to as $qdt$ and is shown to illustrate the flexibility of the model sampling process. It has much larger errors, about $+ - 0.25$ seconds, but is one quarter the size (83Kb) and is therefore more suitable for quick travel time estimates, classroom exercises and web based applications where size and loading speed are more critical than accuracy. Note the factor of 10 change in scale from the previous figure.
Figure 2.3: Residual travel time, this code minus $t_{times}$, of the first arriving P wave (top) and first arriving S wave (bottom). The compressional velocity model was used throughout the core, so the first arriving S-wave beyond about 80 degrees is SKS and beyond about 130 degrees is SKIKS. The small high frequency oscillations (such as between 50 and 90 degrees) in the differential times are the result of the linear interpolant, while the larger offsets are believed to be primarily due to error introduced by the earth flattening transform into the $t_{times}$ values.
be desirable to compare the *ttimes* output with our own for a model with an easy analytic solution over a range of distances and phases. Unfortunately, using different models with the *ttimes* program has proven difficult. The residual for the direct comparison with *ttimes* is shown for the first arriving P wave and S wave. The calculations are for the IASP91 model for an event at the surface, and are sampled at 0.1 degree increments. The high frequency variations in residuals, such as are observed between 50 and 90 degrees in the P-wave residuals, are the result of the use of the linear interpolant in TauP. We believe that the clear trend of increasingly negative relative residuals with increasing distance is likely related to the use of an earth flattening transform within the *ttimes* package, an approximation that can become problematical toward the center of the earth. The TauP package avoids this potential complication by working directly in spherical coordinates. To test this assertion, we compared *ttimes* and TauP results to travel-times calculated by numerical integration of vertical rays of PcP, PKIKP, and PKiKP through the IASP91 model. In each case, our results were closer to the numerical integration results than *ttimes* with the *ttimes* error increasing with increasing depth of penetration. The maximum difference of 0.01 sec for TauP compared to 0.05 sec for *ttimes* for the vertical PKIKP ray accounts for the relative P residual shown in Figure 2.3. Thus, we expect that our solutions have a small, but possibly significant, increase in accuracy relative to *ttimes*, in spite of the current simple linear interpolation.

### 2.3 Seismic Phase Name Parsing

A major feature of the TauP Toolkit is the implementation of a phase name parser that allows the user to define essentially arbitrary phases through the earth. Thus, the TauP Toolkit is extremely flexible in this respect since it is not limited to a pre-defined set of phases. Phase names are not hard-coded into the software, rather the names are
interpreted and the appropriate propagation path and resulting times are constructed at run time. Designing a phase-naming convention that is general enough to support arbitrary phases and easy to understand is an essential and somewhat challenging step. The rules that we have developed are described here. Most of phases resulting from these conventions should be familiar to seismologists, e.g. pP, PP, PcS, PKiKP, etc. However, the uniqueness required for parsing results in some new names for other familiar phases.

In traditional “whole-earth” seismology, there are 3 major interfaces: the free surface, the core-mantle boundary, and the inner-outer core boundary. Phases interacting with the core-mantle boundary and the inner core boundary are easy to describe because the symbol for the wave type changes at the boundary (i.e. the symbol P changes to K within the outer core even though the wave type is the same). Phase multiples for these interfaces and the free surface are also easy to describe because the symbols describe a unique path. The challenge begins with the description of interactions with interfaces within the crust and upper mantle. We have introduced two new symbols to existing nomenclature to provide unique descriptions of potential paths. Phase names are constructed from a sequence of symbols and numbers (with no spaces) that either describe the wave type, the interaction a wave makes with an interface, or the depth to an interface involved in an interaction. Figure 2.4 shows examples of interactions with the 410km discontinuity using our nomenclature.
Figure 2.4: Stylized raypaths for several possible interactions with the 410km discontinuity to illustrate the use of the TauP phase naming convention. A). Top-side reflections. In all cases, the upgoing phase could be either upper or lower case since the direction is unambiguously defined by the “v” symbol, although lower case is recommended for clarity. B). Transmitted phases. In these cases, there are no alternate forms of the phase names since the symbol case defines the point of conversion from P to S. C). Underside reflections from a surface source. The “ˆ” symbol indicates that the phases reflect off the bottom of the 410. Final wavetype symbol must be upper case, since lower case wavetype symbols are strictly upgoing. D). Interactions for a source depth below the interface. Note that P410s converts on the receiver side just as it does for a surface source in B). However, for a deep source, the phase P410S does not exist since a downgoing P-wave cannot generate a downgoing transmitted S-wave at an interface below the discontinuity.
1. Symbols that describe wave-type are:

- **P**: compressional wave, upgoing or downgoing, in the crust or mantle
- **p**: strictly upgoing P wave in the crust or mantle
- **S**: shear wave, upgoing or downgoing, in the crust or mantle
- **s**: strictly upgoing S wave in the crust or mantle
- **K**: compressional wave in the outer core
- **I**: compressional wave in the inner core
- **J**: shear wave in the inner core

2. Symbols that describe interactions with interfaces are:

- **m**: interaction with the moho
- **g**: appended to P or S to represent a ray turning in the crust
- **n**: appended to P or S to represent a head wave along the moho
- **c**: topside reflection off the core mantle boundary
- **i**: topside reflection off the inner core outer core boundary
- **^**: underside reflection, used primarily for crustal and mantle interfaces
- **v**: topside reflection, used primarily for crustal and mantle interfaces
- **diff**: appended to P or S to represent a diffracted wave along the core mantle boundary
- **kmps**: appended to a velocity to represent a horizontal phase velocity (see 10 below)

3. The characters **p** and **s** always represent up-going legs. An example is the source to surface leg of the phase **pP** from a source at depth. **P** and **S** can be turning waves, but always indicate downgoing waves leaving the source when they are the first symbol in a phase name. Thus, to get near-source, direct P-wave arrival times, you need to specify two phases **p** and **P** or use the “times compatibility phases” described below. However, **P** may represent a upgoing leg in certain cases. For instance, **PcP** is allowed since the direction of the phase
is unambiguously determined by the symbol $c$, but would be named $P_c p$ by a purist using our nomenclature.

4. Numbers, except velocities for km$^s_p$ phases (see 10 below), represent depths at which interactions take place. For example, $P_{410s}$ represents a P-to-S conversion at a discontinuity at 410 km depth. Since the S-leg is given by a lower-case symbol and no reflection indicator is included, this represents a P-wave converting to an S-wave when it hits the interface from below. The numbers given need not be the actual depth, the closest depth corresponding to a discontinuity in the model will be used. For example, if the time for $P_{410s}$ is requested in a model where the discontinuity was really located at 406.7 kilometers depth, the time returned would actually be for $P_{406.7s}$. The code “taup_time” would note that this had been done. Obviously, care should be taken to ensure that there are no other discontinuities closer than the one of interest, but this approach allows generic interface names like “410” and “660” to be used without knowing the exact depth in a given model.

5. If a number appears between two phase legs, e.g. $S_{410P}$, it represents a transmitted phase conversion, not a reflection. Thus, $S_{410P}$ would be a transmitted conversion from S to P at 410 km depth. Whether the conversion occurs on the down-going side or up-going side is determined by the upper or lower case of the following leg. For instance, the phase $S_{410P}$ propagates down as an S, converts at the 410 to a P, continues down, turns as a P-wave, and propagates back across the 410 and to the surface. $S_{410p}$ on the other hand, propagates down as a S through the 410, turns as an S, hits the 410 from the bottom, converts to a p and then goes up to the surface. In these cases, the case of the phase symbol (P vs. p) is critical because the direction of propagation (upgoing or downgoing) is not unambiguously defined elsewhere in the phase name. The
importance is clear when you consider a source depth below 410 compared to above 410. For a source depth greater than 410 km, $S_{410p}$ technically cannot exist while $S_{410p}$ maintains the same path (a receiver side conversion) as it does for a source depth above the 410.

The first letter can be lower case to indicate a conversion from an up-going ray, e.g. $p_{410S}$ is a depth phase from a source at greater than 410 kilometers depth that phase converts at the 410 discontinuity. It is strictly upgoing over its entire path, and hence could also be labeled $p_{410s}$. $p_{410S}$ is often used to mean a reflection in the literature, but there are too many possible interactions for the phase parser to allow this. If the underside reflection is desired, use the $p^\wedge_{410S}$ notation from rule 7.

6. Due to the two previous rules, $P_{410P}$ and $S_{410S}$ are over specified, but still legal. They are almost equivalent to $P$ and $S$, respectively, but restrict the path to phases transmitted through (turning below) the 410. This notation is useful to limit arrivals to just those that turn deeper than a discontinuity (thus avoiding travel time curve triplications), even though they have no real interaction with it.

7. The characters $\wedge$ and $v$ are new symbols introduced here to represent bottom-side and top-side reflections, respectively. They are followed by a number to represent the approximate depth of the reflection or a letter for standard discontinuities, m, c or i. Reflections from discontinuities besides the core-mantle boundary, c; or inner-core outer-core boundary, i, must use the $\wedge$ and $v$ notation. For instance, in the TauP convention, $p^\wedge_{410S}$ is used to describe a near-source underside reflection.

Underside reflections, except at the surface (PP, $sS$, etc.), core-mantle boundary ($PKKP$, $SKKKS$, etc.), or outer-core-inner-core boundary ($PKIIKP$, $SKJJKS$, etc.),
SKIIPS, etc.), must be specified with the ^ notation. For example, P^ 410P and P^ mP would both be underside reflections from the 410km discontinuity and the Moho, respectively.

The phase PmP, the traditional name for a top-side reflection from the Moho discontinuity, must change names under our new convention. The new name is PvmP or Pvmp while PmP just describes a P-wave that turns beneath the Moho. The reason the Moho must be handled differently from the core-mantle boundary is that traditional nomenclature did not introduce a phase symbol change at the Moho. Thus, while PcP makes sense since a P-wave in the core would be labeled K, PmP could have several meanings. The m symbol just allows the user to describe phases interaction with the Moho without knowing its exact depth. In all other respects, the ^-v nomenclature is maintained.

8. Currently, ^ and v for non-standard discontinuities are allowed only in the crust and mantle. Thus there are no reflections off non-standard discontinuities within the core, (reflections such as PKKP, PKiKP and PKIIKP are still fine). There is no reason in principle to restrict reflections off discontinuities in the core, but until there is interest expressed, these phases will not be added. Also, a naming convention would have to be created since “p is to P” is not the same as “i is to I”.

9. Currently there is no support for PKPab, PKPbc, or PKPdf phase names. They lead to increased algorithmic complexity that at this point seems unwarranted. Currently, in regions where triplications develop, the triplicated phase will have multiple arrivals at a given distance. So, PKPab and PKPbc are both labeled just PKP while PKPdf is called PKIKP.

10. The symbol km/s is used to get the travel time for a specific horizontal phase velocity. For example, 2km/s represents a horizontal phase velocity of 2 kilome-
ters per second. While the calculations for these are trivial, it is convenient to have them available to estimate surface wave travel times or to define windows of interest for given paths.

11. As a convenience, a \textit{ttimes} phase name compatibility mode is available. So \textit{ttp} gives you the phase list corresponding to P in \textit{ttimes}. Similarly there are \textit{ttt}, \textit{ttp+}, \textit{ttt+}, \textit{ttbasic} and \textit{ttall}.

2.4 Velocity Model Descriptions

This version of the TauP package supports two types of velocity model files. Both are piecewise linear between given depth points. Support of cubic spline velocity models would be useful and may be implemented in a future release.

The first format is the “tvel” format used by the most recent \textit{ttimes} code (Kennett \textit{et al.}, 1995). This format has two comment lines, followed by lines composed of depth, Vp, Vs, and density, all separated by whitespace. TauP ignores the first two lines and reads the remaining ones.

The second format is based on the format used by Xgbm (Davis and Henson, 1993). We refer to it as a “named discontinuity” format. Its biggest advantage is that it can specify the location of major boundaries in the earth. It is our preferred format. The format also provides density and attenuation fields, which will more easily accommodate the calculation of synthetic seismograms in the future.

The distribution comes with several standard velocity models. Users can create their own models by following examples in the \textit{User’s Guide} included in the distribution package. Standard models are: IASP91 (Kennett and Engdahl, 1991), PREM (Dziewonski and Anderson, 1981), AK135 (Kennett \textit{et al.}, 1995), Jeffries-Bullen (Jeffreys and Bullen, 1940), 1066a and 1066b (Gilbert and Dziewonski, 1975), PWDK (Weber and Davis, 1990), SP6 (Morelli and Dziewonski, 1993), Herrin (Herrin, 1968).
2.5 Utilities in the TauP Toolkit

There are 8 separate utilities in the TauP Toolkit. Each is described in detail in the User’s Guide, so descriptions here are brief. Figure 2.5 summarizes the outputs of several of the codes.

1. **taup_create.** This utility takes a velocity model in one of the two supported formats and does the sampling and branch integration processes to produce a TauP model file for use by all other utilities.

2. **taup_time.** This is the TauP replacement for the *ttimes* program. At a minimum, given phase, distance and depth information, it returns travel times and ray parameters. Options allow for station and event locations to be provided in lieu of distance as well as for more specialized outputs.

3. **taup_curve.** Produces entire travel time curves for given phases. Options include the ability to output these curves in a format suitable for plotting with GMT (*Wessel and Smith*, 1991).

4. **taup_path.** Calculates ray paths for given phases. Options include the ability to output these paths in a format suitable for plotting with GMT (*Wessel and Smith*, 1991).

5. **taup_pierce.** Calculates piercing points at model discontinuities and at specified depths for given phases. Options include the ability to output these points in a format suitable for plotting with GMT (*Wessel and Smith*, 1991).

6. **taup_setsac.** Utility to fill SAC (*Tull*, 1989) file headers with theoretical arrival times.

7. **taup_table.** Utility to generate travel time tables needed for earthquake location programs. Currently, only an ASCII table format is supported along with
Figure 2.5: Examples of output from the TauP Toolkit utilities. Some modifications in a touch-up program were done to prepare the figures for publication. A) Output of taup_path for the phase $s^{*}410PS^{*}670PPms$ at a distance of 155 degrees from an event at 630 km depth. $s^{*}410PS^{*}670PPms$ arrives from both directions, one ray traveling 155 degrees, the other 205 degrees. The model PREM is used. Source is given by the *, station by the triangle. B) Output of taup_curve for the same phase as in A). A reducing velocity of 0.125 deg/s was used for this plot. Travel-time curves for PPP, SS, and SKKS are shown for reference. C) Output of taup_pierce for the same phase as A) was added to an existing GMT script to produce this output. Points of interaction with the 400km discontinuity are shown for a hypothetical event near East Pacific rise arriving at a hypothetical station in central India. The path for the ray that travels 155 degrees to the station is shown in a solid line, interaction points with the 400km discontinuity are solid diamonds. The 205 degree raypath is shown as a dashed line with solid circles indicating interaction points at 400km depth.
a generic output. Other formats could be supported if interest warrants.

8. **taup.peek**. Debugging code to examine contents of a taup velocity model file.

The TauP Toolkit is written entirely in the Java programming language. This should facilitate use of the utilities on any major computer platform. It has been tested on Solaris UNIX, MacOS, and Windows95. The distribution includes simple scripts to facilitate use of the above utilities in a UNIX environment. It also provides mechanisms for accessing the utilities via C programs and TCL (via the “jacl” implementation) scripts. At present, only raw Java command line interfaces exist, limiting the usefulness in MacOS and Windows environments. A WWW based applet interface was tested by our undergraduate geophysics course in the Spring of 1998, however a more refined GUI is planned for development soon which should make the package much more useful in non-UNIX environments. The package can be obtained from http://www.seis.sc.edu/taup.

### 2.6 Acknowledgments

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Chapter 3

Compression

3.1 Abstract

With the larger volumes and growing use of near real time data transmission, improved compression of seismic data is increasingly important. The algorithms in use within the seismological community do not take advantage of current techniques within the larger computer science community nor do they take advantage of all of the properties of seismic time series. We introduce a new scheme for the loss-less compression of seismic data drawing upon general purpose compression techniques as well as analysis of the properties of seismic time series. Compression of time series can be effectively broken into two independent processes operating in series. The first is a prediction process that transforms the original time series into an equivalent time series of prediction residual whose values are more closely clustered around zero than the original. The best balance of quality and speed seems to be first, second and third differences, with the choice depending on the actual data to be compressed. The second process is an efficient encoding of the residuals from the prediction process. Huffman encoding is one of two widely used methods in the computer science community, and is simpler to implement and understand than arithmetic encoding. Current compression algorithms for seismic data concentrate on the encoding process,
with little flexibility given to prediction. We propose that a wider choice of prediction operators can have a major impact on compression ratios, and existing compression algorithms could benefit with minimal change by allowing second and third differencing. Huffman encoding provides an attractive and efficient encoding scheme, and has the additional advantage of being widely accepted and used within the broader computational community.

3.2 Introduction

The volume of seismic data collected and archived has grown tremendously since the advent of digital recording and rate of growth is increasing. The IRIS Data Management Center currently archives in excess of 10 terabytes of data per year and is likely to only increase. At these levels of data flow, even moderate increases in effectiveness of data compression can have a significant on the required archive storage space.

Current algorithms widely used within seismology include the Steim1, Steim2 and US National Network compression. Each of these specifies a first differencing scheme followed by an encoding with keys and values where the key specifies how many bits the value uses. These are generally effective at reducing the storage size of data.

3.3 Entropy

A useful concept from information theory is that of entropy. Entropy is a measurement of how much information a data stream carries. One way to define this is to separate the signal into two parts, a predictable part and an unpredictable part. The predictable part does not need to be transmitted in order for the receiver to reconstruct the original signal. The unpredictable part, on the other hand, represents the disorder in the data stream and must be transmitted. Entropy is the measure of
disorder in a thermodynamic system and so the same term is used to measure this unpredictable part, which is the information content of a data stream. It may seem contradictory at first to equate disorder with information content, however, a data stream that is perfectly predictable carries no information and has no disorder. In a completely unpredictable data stream, by contrast, every single bit is needed to reconstruct the input values.

Entropy is expressed as the average number of bits needed, per sample, to recover the original signal. This establishes a lower bound on the size of any compressed version of the data and thus gives a mile-post to compare effectiveness of compression algorithms. In the case of a signal where each sample is independent of the previous sample, the entropy can be defined as

\[ E = - \sum_{i \in N} p_i \log_2(p_i) \] (3.1)

where the sum is over all symbols in the alphabet. The alphabet for a particular stream of seismic data, for instance, might be all numbers expressible as 32 bit integers. It is more complicated in the case where the next sample has a complicated dependence on the previous samples. Knowing the entropy, the redundancy within the data stream can be defined by

\[ R = \log_2(|N|) - E \] (3.2)

where \(|N|\) is the size of the alphabet.

Although seismic data almost certainly does have dependence on previous samples even after differencing, it is generally useful to make the assumption of independence. If there was a known relationship between the current value and previous values, we could simply apply the corresponding correction to the predictive operator, and arrive at the assumption of independence. In addition, even if there is dependence,
theoretical best compression for an encoder that cannot make use of the dependence is still bounded by this definition of entropy.

3.4 Compression

The compression of many types of time series in general, and seismic data in particular, can be modeled as the combination of two processes, a predictive operator and a mechanism for efficiently encoding the residual of the prediction. The residual is hopefully composed of very frequently occurring small values and less frequent large values. It is important to realize that these two operations are independent, the choice of encoding need not determine the choice of prediction.

3.4.1 Prediction

The predictor is a function that, given information on some number of previous samples, returns a prediction of the next sample. This predictor need not return the actual value of the next sample, clearly this is not possible. Rather, it merely needs to have errors, or residuals, that are on average smaller in absolute value than the sample. The decompresser, knowing the predictive operator and the previous samples, can simply perform the inverse operation to recover the true value of the current sample.

The ability of the predictor to concentrate the residual values around zero is very important to achieving a good compression. Existing seismic compression algorithms tend to use a single fixed order of differencing as a simple predictor, for example the first differencing in the widely used Steim1 and Steim2 algorithms. Options for predictive operators include first difference, \( P(n) = f(n - 1) \), as well as the no-differencing or null operator, \( P(n) = 0 \), second difference, \( P(n) = 2f(n - 1) - f(n - 2) \) and third difference, \( P(n) = 3f(n - 1) - 3f(n - 2) + f(n - 3) \). There are of course
others of higher orders. We have found that prediction should be allowed to vary based on properties of channel for best results. In fact in many cases, second differences can be as big an improvement over first differencing as first differencing is over no differencing. Third differences can be better than second for some channels.

The residual of a predictor operating on a time series is also a time series. And in fact, it can be of the same length as the input if a sufficient number of zeros are prepended to account for the order of differencing. This eliminates any need for storing integration constants in a special way at the beginning of the compressed stream. For example, if second differencing is used then prepending two zeros to the stream before differencing results in an output stream of the original length. If the original series, prepended with two zeros, is \(0, 0, t_1, t_2, t_3, \ldots\), then the second differenced stream is \(t_1, t_2 - 2t_1, t_3 - 2t_2 + t_1, \ldots\) Notice that nothing special is done for the first two samples. In addition, if \(t_1\) and \(t_2\) are of approximately the same magnitude, no additional space it required as \(t_2 - 2t_1 \approx -t_1\). If this technique is used, any time series encoding, such as Steim1, can be used with any order of differencing, such as second differences, to achieve the best compression for the given input series. One possible flaw can occur if \(t_2 - 2t_1\) is greater than the size of the storage, for example if \(t_2 - 2t_1\) was greater than \(2^{31}\). This is not a problem with current data as they are limited in size due to the commonly used 24 bit digitization.

Deciding which level of differencing to use can be done by computing some basic statistics for a small, typical time series for a channel, and then assuming that the background noise levels and spectral characteristics will not change significantly over time. This is likely reasonable in most cases, but periodic checking is easy and may be wise. In general, the encodings used depend on most of the values being clustered around 0. In other words we can use the minimization of the mean and standard deviation of the residual series as a proxy for compressibility. As a practical matter, we can ignore minimizing the mean, because if the best predictor, \(P\), resulted in a
series with a mean of $\mu = \mu_0 \neq 0$, then we could form a new predictor $P' = P - \mu_0$ which has mean zero. Our experience has been that first, second and third differencing result in a series with very near zero mean. Thus, the computation of the best order of differencing to use reduces to minimizing the standard deviation, which results in a few values around zero with high frequency of occurrence and the rest of the values with a low frequency of occurrence.

There are instances where a drifting sensor may cause a bias in the mean. For instance, suppose a sensor drifts from a mean near zero to a mean near 100,000 over a period of 1000 samples. The average value of the first differences will be 100 since this is the average increase per sample. However the effect is limited to either a series of small differences over a long period of time, or a series of large difference over a short period of time simply because a large rate of increase will quickly peg the sensor. In either case, it is unlikely to have a significant effect on the compression over the long term.

It might seem that if first differencing reduces the standard deviation, then second should be better, and third even better. In fact, a minimum is usually reached very quickly, and the results get progressively worse with higher orders. For example, first differencing of the simple time series shown in the first row of Table 3.1 results in an improvement, with the mean smaller by more than a factor of 10 and the standard deviation dropping from 3.56 to 2.22. Second differencing results in another improvement, with the mean smaller and standard deviation of 1.81. However, subsequent differencing actually results in an increased standard deviation, and hence third differencing is actually a worse predictor than first or second differencing. Conceptually, one can think of differencing as reducing the amplitude of lower frequency signals in the time series while exaggerating the frequencies near the Nyquist. Consider a sine wave sampled exactly at the peaks and troughs, giving a series 1, −1, 1, −1, . . . Differencing in this case would double the amplitude, yielding −2, 2, −2, 2, . . . Clearly
Table 3.1: Effect of three differencing schemes (first, second and third differences) on a sample time series. First order differencing makes a large improvement in mean although with an increase in standard deviation. Second and third differencing are not an improvement in this case, both leading to increased mean and standard deviation over first differences.

the standard deviation is not reduced in this case.

When the time series is dominated by relatively longer period energy, differencing results in a net improvement. Once the short periods come to dominate, further differencing increases the number of large values and hence the standard deviation.

While differencing is the natural first guess at predictive operators, they generalize to a whole class of predictors known as ARIMA (Box and Jenkins, 1976). These were originally created for forecasting business phenomena, but are applicable to a wide range of time series. The generalization involves first applying differencing to the best order possible, and then creating an additional predictor that uses a linear combination of both the previous $n$ samples, as well as the last $n$ predictions or equivalently the last $n$ residuals. The name ARIMA comes from auto-regressive for the use of the last samples, integral for the differencing, and moving average for the use of the residuals.

While this generalization seems to open a wide array of possible predictions, we have not found it to give significant improvements over simply using the best order of differencing. With the increase complication and the difficulties of using floating point operations across different computer architectures, we believe that limiting predictors to differencing is the most practical choice.
3.5 Encoding

Once the predictive operator has reduced the magnitudes of the samples, it remains to efficiently encode them, taking advantage of the fact that the residual has smaller variance. There are various means of doing this, with the current seismological algorithms using flags to identify the number of bits used for the next sample or samples. The encoding of the residual time series values is not something unique to seismology, and hence looking to the domain of computer science is wise. There are two main encoding schemes in wide use for compression in general, Huffman and arithmetic encoding, that both achieve near optimal encoding.

Arithmetic encoding (MacKay, 2003) is the more complicated, although with the advantage of achieving optimal encoding, ie encoding is at the level of entropy. Huffman is simpler and while it only achieves optimality in certain cases, the difference for seismic data is not likely significant. We have chosen to use Huffman codes.

Many good references can be found on Huffman encoding, (Huffman, 1952; MacKay, 2003), and so we only give a brief overview. The basis of Huffman codes is to generate a binary code for each “letter” of a given alphabet such that the most frequently appearing letters receive the shortest codes. It seeks to minimize $\Sigma f_i l_i$ where $f_i$ is the frequency of the ith letter and $l_i$ is the length of the code. It is apparent that the code will achieve optimal compression, ie entropy, if $l_i = \log_2(f_i)$. In other words, if all of the frequencies are integral powers of 2. As this is rarely the case, the outcome will be slightly worse than optimal.

One other important criteria for a successful coding scheme that uses variable length codes is the prefix property. It requires that no code can be a prefix of another code. This is readily seen to be necessary for decompression as it would be impossible to tell whether the code stopped at the end of the prefix, or continued on. What is not so obvious is that the prefix property is also sufficient for decompression.

To use Huffman encoding, we must define the alphabet. The natural alphabet
for seismic data is all integers $i$ with $-2^{23} \leq i \leq 2^{23}$. This is also not practical. Instead, we use Huffman codes for a subrange of this natural alphabet that contains the bulk of the frequently occurring values and then use an escape code to deal with the occasional value outside of that range. We have found ranges of $\pm 2^{10}$ or $\pm 2^{12}$ have reasonable trade offs between size and compressibility. Even for noisy stations, an insignificant number of samples lie outside of this range once an appropriate order of differencing has been applied.

The other requirement for building Huffman codes is the frequency of occurrence of each value within the alphabet. We can assume seismic background noise is very close to a normal distribution, and hence we need only calculate the standard deviation in order to characterize the frequency distribution. This single measurement allows the Huffman codes to be tuned to best compress the time series.

### 3.6 Application to Seismic Data

We have applied our compression algorithm to seismic data from the IRIS DMC. Figure 3.1 shows a histogram of the ratio of the original compression, either Steim1 or Steim2 (Ahern and Dost, 2006), to this algorithm. The majority of the data compresses better with this algorithm than the existing ones. The total size of the original compressed data was 149 Mb. Using our algorithm, the sizes are 137 Mb, 128 Mb and 139 Mb using first, second and third differencing respectively. If instead of picking a single difference order we choose the best order of differencing for each seismogram, the size is 123 Mb, an improvement of over 17%. With this dataset, second differencing is clearly the best single choice, and picking the order of differencing on a per seismogram basis can do even better.
Figure 3.1: Histogram of the ratio existing compression algorithms at the IRIS DMC, mostly Steim1 and Steim2, to this algorithm. The data was all BHZ seismograms at IU stations for all magnitude 6 or larger earthquakes in January of 2006. The request was for 1 hour of data starting 2 minutes before the P arrival, although data availability affects this. The majority of the data compressed better with our algorithm.
3.7 Conclusions

We have found that the choice of predictive operator or equivalently order of differencing is critical to achieving near optimal compression of seismic data. Simply using first differences in a “one size fits all” approach sacrifices the additional compression possible for channels where second, and even third, differences provide very real improvements.

Huffman codes, while more complex at first glance than the traditional seismic encoding schemes, are effective, and achieve near optimal compression. They are a standard within the broader realm of data compression in general, being used in the popular general purpose compression tools. They have the added benefit of being capable of being tuned to best encode the given data stream, taking utmost advantage of streams that have very small variances, while still being able to handle difficult to compress high variance data streams.

Disadvantages of the Huffman codes include the fact that they are not byte aligned, which trades off speed for increased compression. The increased compression ratios seem worth the small speed penalty. They are conceptually more difficult to understand because of the varying length codes, but the algorithm need only be understood in detail by the software engineers that will implement the compression and decompression routines.

One additional difficulty is the need for floating point operations in creating the Huffman codes. While using the actual standard deviation of the stream creates the best codes, the penalty for using a close standard value may not be too great. For example codes could be pre-generated for standard deviations of 4, 8, 16, 32,..., 4096, which cover a wide range from very quite to very noisy channels, while still keeping the codes relatively optimized. This would use an additional 8 of the 64 available compression codes in SEED.

Unfortunately, the current Data Definition Language within SEED (Ahern and
Dost, 2006) is not capable of handling the increased complexity of Huffman codes. This is likely not a significant handicap because it could be extended in a later version, and as a practical matter, it does not seem to be widely used anyway.

It cannot be emphasized enough the importance of the choice of order of differencing in compression of seismic data. Even in cases where Huffman codes are deemed too complex, it would still be highly advantageous to allow second and third order differences to be used with, for example, Steim1 and Steim2. This could be easily handled by using the first 2 bits of the compression byte in SEED to specify the order of differencing. Backwards compatibility could be achieved by using 00 for the existing definitions, with 01, 10 and 11 for first, second and third differences, respectively.
Chapter 4

EARS

4.1 Introduction

The EarthScope Automated Receiver Survey, EARS, aims to calculate bulk crustal properties for all broadband, three component, seismic stations, current and historical, within the continental United States that are available from the IRIS DMC (Crotwell and Owens, 2005, 2006). EARS uses the $H_\kappa$ receiver function stacking technique of Zhu and Kanamori (2000). These results will be distributed to interested research and education communities as a “product” of the USArray component of EarthScope (EarthScope, 2007). Because of the high level of automation that this requires, a natural extension of this effort is to calculate similar results for all stations in the IRIS DMC and to date, the EARS database contains over 180,000 receiver functions for over 1700 stations from around the world. To do this, we have employed SOD (Owens et al., 2004), a FISSURES/DHI-based software package (Ahern, 2001a,b) that we have developed to do automated data retrieval and processing.

EARS is an example of what we term receiver reference models, (RRM). These are analogous to the Harvard Centroid Moment Tensor solutions (which could be termed source reference models). An RRM need not be a definitive result but rather provides a standard, well known, globally consistent reference. This may be sufficient for many
users, just as CMT’s are sufficient for many, but may also be a starting point for more in-depth, focused studies. The key features of an RRM are that it be generated from a well known and widely accepted technique, produce results for most or all stations, and provide updated results as new sources of data become available. In addition, it must produce results that are of interest and in a form usable by the community.

Because automated processing proceeds, by definition, without a large amount of human input and guidance, there are differences between it and traditional seismic processing. An automated system must rely on quantitative measures of quality as opposed to the seismologist’s insight into what constitutes a “good” or “bad” earthquake or seismogram. The advantage of non-human-driven processing is that the volume of data to be used can be substantially larger. Use of these larger data volumes can have quality implications as well, making use of stacking techniques and statistics instead of intuition and detailed analysis by a seismologist. If a seismic technique can be structured so that it can be driven by SOD, then there is really no reason not to apply it to as large a volume of data as possible. Hence, we envision the RRM as just the first of many automated products, created by us and by others.

All data and results for EARS are available at http://www.seis.sc.edu/ears.

4.1.1 Processing Overview

The development of SOD, Standing Order for Data (Owens et al., 2004), has provided the data access and processing framework on which EARS was built. SOD handles the searching, retrieval and preprocessing of the data, while EARS needed to add just the receiver function specific items (Figure 4.1). SOD is responsible for the initial data retrieval. This includes event and station selection, retrieving the waveforms, preprocessing and finally driving the receiver function calculations. The calculation of the receiver functions is done with a special purpose SOD processor and is based on the iterative deconvolution technique of Ligorría and Ammon (1999).
The finished receiver functions are then stored in the database and single-event receiver function $H\kappa$ stacks are calculated, making use of the phase weighted stacking method of *Schimmel and Paulssen* (1997). Automated quality control steps are also performed at this point, and will mark low quality results as rejected. On a periodic basis full $H\kappa$ stacks based on all successful events at each station are computed. From these multi-receiver function $H\kappa$ stacks, the best crustal thickness and $V_p/V_s$ are extracted, along with bootstrapped error estimates and the complexity measurement of the quality of the result.
Figure 4.1: The EARS processing system. SOD handles the event and channel selection, waveform retrieval and preprocessing of the seismograms (upper left). SOD then drives the iterative receiver function calculation via a custom processor (upper right). The results are then sent to the central database where results are stored and the $H\kappa$ stacking takes place.
4.2 Receiver Functions

Receiver function analysis is a well known technique for estimating bulk crustal structure beneath a seismic station and has been used and refined by many authors over the years (see, for example Langston, 1979; Owens et al., 1984; Ammon et al., 1990; Ammon, 1991; Ligorría and Ammon, 1999; Zhu and Kanamori, 2000). Receiver functions determine near receiver earth structure by the strength and timing of S wave reverberations generated by a P wave as it interacts with layer boundaries. They are defined as the spectral division of the radial by the vertical component. In equation 4.1, $H(\omega)$ is the Fourier transform of the receiver function, $S(\omega)$ is the source time function, $R(\omega)$ and $Z(\omega)$ are the Fourier transforms of the radial and vertical impulse response of the near receiver structure (Ammon, 1991). Note that the radial and vertical components share the same source time function, thus canceling its effect within the spectral division.

$$H(\omega) = \frac{S(\omega)R(\omega)}{S(\omega)Z(\omega)}$$

One useful property of the receiver function is that there is a correspondence between individual peaks and troughs on the receiver function and arrivals as recorded on the radial seismogram (Ammon, 1991). With the exception of the direct P wave, the receiver function deconvolution effectively removes all phases with a final P wave leg. Phases with a final S wave leg, in contrast, are not removed by the deconvolution, and appear as arrivals in the receiver function. There are five reverberations with significant energy that appear in a receiver function generated by a single simple discontinuity. They are denoted by $P_s$, $P_pP_s$, $P_sP_s$, $P_pS_s$ and $P_sS_s$ (Figure 4.2). The notation is that the initial capital P is the P wave from the source to the base of the Moho, or other discontinuity, and each additional letter represents a leg within the crust with lowercase corresponding to up-going legs and uppercase corresponding to
down-going legs. In the case of the PsPs and PpSs reverberations, the arrival time is the same as they have the same number of P and S wave legs, just reordered, and are often labeled as PpSs/PsPs or just PsPs. The last reverberation, PsSs, is significantly smaller than the others, and so is not used in the $H_\kappa$ stacking technique.

![Ray paths for P, Ps, PpPs, PsPs and PpSs](image1)

Figure 4.2: Ray paths for P, Ps, PpPs, PsPs and PpSs

![Synthetic receiver function](image2)

Figure 4.3: Synthetic receiver function
4.3 Iterative Deconvolution

Historically, receiver function calculations, being a deconvolution, were implemented by division in the frequency domain. Unfortunately, division in the frequency domain has inherent stability issues (Ammon, 1991) largely related to spectral holes, places where the spectrum of the denominator becomes very small. Generally, a technique such as water level deconvolution (Clayton and Wiggins, 1976) is used to stabilize the processing. However, this technique is difficult to use in an automated fashion as the appropriate water level is found via trial and error with an analyst making a visual determination of whether the bounds of stability have been crossed. Instead, we have used the iterative deconvolution technique of Ligorría and Ammon (1999) which does not suffer from these stability issues and is hence much easier to use in an automated system.

The result of the deconvolution by spectral division is a time series that when convolved with the denominator best matches the numerator. The time series can then be thought of as a sum of weighted delta functions. At each point in the deconvolution where the amplitude is large in absolute value, there is a high correlation or anticorrelation between the numerator and denominator. Thus, the deconvolution process can be approximated by repeatedly finding the best correlation between numerator and denominator over all time lags and subtracting the convolution of this spike with the denominator from the numerator. The iterative deconvolution replaces the spectral division with a repeated process of subtracting the best correlation. Because this technique tends to pick out points of large correlation and anticorrelation first, it is easier to retrieve the useful information in the deconvolution without suffering from the instabilities of spectral holes found in frequency domain deconvolution.

Our implementation of the iterative deconvolution is a Java translation of the iterDecon Fortran code of Ligorría and Ammon (1999). It begins with the numerator and denominator seismograms, usually the horizontal and vertical respectively, with
all preprocessing such as filtering, instrument correction and trend and mean removal already applied. After each seismogram is padded with zeros to the next larger power of two, a Gaussian filter is applied to both, generally with width between .5 and 5, depending on the desired frequency content. Within EARS we have chosen the Gaussian width to be 2.5 being a good balance between the precision of the thickness and $V_p/V_s$ results and being able to resolve boundaries in the presence of noise. We have also investigated some stations with a Gaussian width of 1.0.

Within each iteration, the correlation between the current numerator and the denominator is calculated. The index of the largest absolute value of the correlation defines the lag at which the denominator best matches the numerator. A negative correlation can occur at an arrival with a 180 degree phase shift such as PsPs. An impulse with amplitude set to the correlation, normalized by the zero lag autocorrelation of the denominator, is added to the series of spikes at the corresponding lag time. A time series, constructed from the impulses, is then Gaussian filtered and convolved with the denominator to form a prediction of the numerator. The prediction is then subtracted from the numerator. If the power in the numerator, scaled by the denominator, is small enough, the iteration stops. Otherwise it continues until the maximum number of repetitions is reached. The resulting Gaussian filtered time series is the receiver function.
Figure 4.4: The location of stations used in examples.
Figure 4.5: Record section of the 46 radial receiver functions for station TA.S08C, see Figure 4.4 for location. The direct P corresponds to time 0, while the Ps, PpPs and PsPs/PpSs reverberations arrive at about 5, 17 and 22 seconds, respectively. The time scale is the lag time in the deconvolution.
4.4 Determining Crustal Structure

Because there is the correspondence between P-to-S wave phases and the peaks and troughs on the receiver function, the bulk properties of the crust can be determined from the relative time lag of these arrivals on the receiver function. There is a trade off between thickness of the crust ($H$) and the average $V_p/V_s$ of the crust (Ammon et al., 1990). However, each of the reverberations has a different trade off relationship, and hence in principle by analyzing all three phases simultaneously the correct depth and velocity ratio can be determined.

It has been shown that the timing of the reverberations is largely insensitive to absolute P and S velocities in the layer, and the best that can be done is to extract the thickness and the ratio of the P to S wave velocity (Ammon et al., 1990). Therefore a P velocity is usually assumed, and the best combination of crustal thickness and $V_p/V_s$ ratio is calculated.

The relative arrival times of the three reverberations with respect to the direct P wave, for a simple layer over a half space, are given by:

\begin{align}
   t_{Ps} - t_P &= H(\eta_\beta - \eta_\alpha) \\
   t_{PpPs} - t_P &= H(\eta_\beta + \eta_\alpha) \\
   t_{PsPs/PpSs} - t_P &= 2H\eta_\beta
\end{align}

where $\eta_\alpha = \sqrt{1/V_p^2 - p^2}$ and $\eta_\beta = \sqrt{1/V_s^2 - p^2}$ are the P and S wave vertical slowness respectively and $p$ is the ray parameter.

If the receiver functions are of high quality, then it is possible to simply pick the arrival times of the reverberations and find a best fit of crustal properties (e.g. Zandt and Ammon, 1995). However, in making use of noisier stations atop complex structure recording smaller earthquakes, it can be difficult if not impossible to discern these arrivals by eye. In this case, looking at receiver functions from many events at...
a given station is desirable, which leads to use of a stacking technique.

The simplest stacking technique is to sum the receiver functions. Unless the events are all at similar distances, this is unlikely to generate good results as the timing of the reverberations has a dependence on the ray parameter \( p \), which depends on the distance. A slant stack can be used to account for the ray parameter dependence. A further stacking idea of Zhu and Kanamori (2000) is to remove the ray parameter dependence by transforming the receiver function, as amplitude as a function of time, to a \( H\kappa \) stack, with amplitude as a function of thickness, \( H \), and \( \kappa = V_p/V_s \). Once this transformation is done, there is no longer a ray parameter dependence, and \( H\kappa \) stacks from events at many distances can be stacked. The \( H \) and \( \kappa \) corresponding to the peak amplitude can be interpreted as the single layer model that best matches the receiver functions. There is a distinction between an \emph{intra-seismogram} \( H\kappa \) stack that stacks the three phases from a single receiver function, and a \emph{multiple-seismogram} \( H\kappa \) stack that stacks all three phases from many receiver functions. While there is not a difference in the construction, other than more receiver functions being involved, the \emph{intra-seismogram} \( H\kappa \) stack uses only energy from a single earthquake. This makes it more sensitive to noise or poor signal from the event. But it does give useful information about the quality of the event and can lead to the discovery of problem data. For actual crustal property determination, it is best to use a \( H\kappa \) stack that uses many earthquakes so that the stacking technique has sufficient data to amplify coherent signal and downweight incoherent signal.

The transformation of the receiver function from the time domain to the \( H\kappa \) domain is done by summing amplitudes at the appropriate predicted times for a range of models. For each \((H, \kappa)\) pair, a single layer model is created, and from this the arrival times of the various phases relative to the P for a given ray parameter are calculated using Equation 4.2. Once these times are known, the \( H\kappa \) stack, \( s(H, \kappa) \),
is given by
\[ s(H, \kappa) = \sum_{k=1}^{N} \left( w_1 r_k(t_{Ps}) + w_2 r_k(t_{PpPs}) - w_3 r_k(t_{PsPs}) \right) \] (4.3)

where \( r_k(t) \) is the \( k^{th} \) radial receiver function, \( t_{Ps} \) is the relative arrival time of the \( Ps \) phase, respectively \( PpPs \) and \( PsPs \), and \( w_i \) are weights based on the seismic ray path. The negative sign on the \( PsPs \) arrival is due to the reversed polarity of this phase. In EARS, we take all the arrival weights to be \( 1/3 \) but other authors have chosen differently. Zhu and Kanamori (2000) choose \( w_1 = 0.7, w_2 = 0.2, \) and \( w_3 = 0.1 \) arguing that the later two arrivals are smaller and so should contribute less. Dugda et al. (2005) also choose unequal weighting with values of 0.6, 0.3, 0.1 respectively. Given the highly automated nature of our processing, and the lack of strong arguments favoring one weighting over another, we have chosen to allow the weights to remain equal, favoring no phase over another.

One issue with the \( H\kappa \) stacking is that while the stack should be relatively insensitive to the \( V_p \), a value still is needed. The sensitivity, on the order of 0.5 kilometer change in thickness for a 0.1 kilometer per second change in \( V_p \) (Zhu and Kanamori, 2000), is small, but it still is worth trying to use a reasonable value. With EARS being highly automated, it requires an estimate of \( V_p \) for all stations across the globe. Thus, we have chosen to use the \( V_p \) estimate from the Crust2.0 global crustal model (Bassin et al., 2000; Laske et al., 2007). This gives a uniformity to the global estimates while still allowing for known variations in local crustal structure. The value is chosen to be the average \( V_p \) as seen by a normal incident wave for the \( 2^\circ \times 2^\circ \) cell to which the station belongs.
4.5 Phase Weighted Stacking

One problem with any type of stacking of time series is that a large amplitude on a single or a few time series can create a noticeable signal in the resulting stack, even if the most of the remaining time series do not contain a corresponding arrival. Generally with stacking, one is interested in signals that, while weak, are consistent across all or most of the time series, and so down-weighting signals that appear on only a few traces is desirable. We make use of the phase weighted stacking technique of Schimmel and Paulssen (1997) when computing the $H\kappa$ stacks because it offers a good ability to extract consistent weak signals while offering low distortion, resulting in sharper $H\kappa$ stacks with less noise. Phase weighted stacking is a nonlinear stacking technique that can improve the strength of coherent signals. The weights are based upon the instantaneous phase of the time series at the given point in time and more weight is given to signals with consistent phase.

4.5.1 Analytic Signal

In constructing a phase weighted stack, the real valued receiver function, $r(t)$, is converted into a complex analytic signal, $R(t)$ (Bracewell, 1978). The receiver function is assigned to the real part of $R(t)$ and the Hilbert transform of $r(t)$, $H(r(t))$, is assigned to the imaginary part, so $R(t) = r(t) + iH(r(t))$. The analytic form of a signal effectively throws away the negative frequencies. Thus, the analytic form of a signal can also be constructed via the Fourier transform by setting amplitude of all negative frequencies to zero, doubling the positive frequency amplitudes and leaving the zero frequency amplitude unchanged. The conversion of a time series to an analytic signal does not lose information as the negative frequencies are recoverable due to the symmetry properties of the Fourier transform, and the analytic signal is often useful in extracting certain properties that can be obscured in the original series.
Because the analytic signal is complex valued, it can be written in terms of amplitude and phase, 
\( R(t) = A(t)e^{i\Phi(t)} \). The phase, \( \Phi(t) \), is termed the \textit{instantaneous phase} of
the real signal \( r(t) \) and this is used in the phase weighted stack. The amplitude \( A(t) \) is known as the envelope and provides an instantaneous amplitude. It should
be emphasized that this instantaneous phase is different from the usual concept of phase. The instantaneous phase is phase as a function of time, whereas in the Fourier
transform sense phase is a function of frequency. Just as the envelope function provides a local amplitude at a point in time, instantaneous phase provides information on the local phase of the signal at a point in time, and can thus be used as a measure of the coherency of the wave shape across many time series.

4.5.2 Phase Weight

We wish to make use of the instantaneous phase by modifying the usual linear stack, 
\( g(t) = \frac{1}{N} \sum_{j=1}^{N} r_j(t) \), to have a weight on a per sample basis based on the instantaneous phase. The weight, as a function of time, is the average of the normalized complex values, optionally raised to a power \( \nu \), thus \( w(t) = \left| \frac{1}{N} \sum_{k=1}^{N} e^{i\Phi_k(t)} \right|^\nu \). Thus, the instantaneous phase from each trace has equal weight regardless of the amplitude of the individual trace at that point as there is no amplitude dependence on the weighting, aside from the initial calculation of the Hilbert transform. By weighting the stack with the sum of unit vectors in the complex plane, amplitudes of signals with coherent phase are preserved, with weight near 1, while signals with wildly varying instantaneous phase will tend to sum destructively yielding a weight near zero. The final form of the phase weighted stack, \( g_{pw}(t) \), is then

\[
g_{pw}(t) = \left( \frac{1}{N} \sum_{j=1}^{N} r_j(t) \right) \left| \frac{1}{N} \sum_{k=1}^{N} e^{i\Phi_k(t)} \right|^\nu \tag{4.4}
\]
4.5.3 Application to $H\kappa$ Stacking

While phase weighted stacking is generally applied to stacking of time series, it is easily used within the $H\kappa$ technique. The ordinary $H\kappa$ stack is constructed by Equation 4.3 and to incorporate the phase weighted stack, we append a multiplicative weight that is the phase weight. We calculate the analytic signal $R(t) = A(t)e^{i\Phi(t)}$ and then apply the weighting to the $H\kappa$ stack to get, setting all arrival weights to $1/3$:

$$s_{pw}(H, \kappa) = \frac{1}{3} \sum_{k=1}^{N} \left( r(t_{Ps}) + r(t_{PpPs}) - r(t_{PsPs}) \right)$$

$$\left| \frac{1}{3N} \sum_{k=1}^{N} \left( e^{i\Phi_k(t_{Ps})} + e^{i\Phi_k(t_{PpPs})} - e^{i\Phi_k(t_{PsPs})} \right) \right|$$

Note the negative on the phase weight of $PsPs$ as the polarity reversal in the $PsPs$ arrival causes the instantaneous phase to be of opposite sign.

A comparison of $H\kappa$ stacks with and without phase weights are shown in Figure 4.6 for station TA.S08C, see Figure 4.4 for the station location. This shows only a subset of the $H\kappa$ stack around the maximum, the full $H\kappa$ stack is shown in Figure 4.13. In general, phase weighting sharpens the peak around a maximum, resulting in a more precise estimate of thickness and $V_p/V_s$. It can also reduce the amplitude of a local maximum if the signal is not coherent across the stacked receiver functions, which might result in a different local maximum being the global maximum.
Figure 4.6: Phase weighting versus no phase weighting at TA.S08C. The top shows a cross section at $\kappa = 1.7675$ on the two $H\kappa$ stacks below (red and blue lines). This passes through the global maximum of the phase weighted stack. Using phase weighting is shown in blue and in the bottom $H\kappa$ stack. Without phase weighting is shown in red and in the middle $H\kappa$ stack. The peak in the non-phase weighted stack is significantly broader, mostly due to the side lobe, which is damped down in the phase weighted stack. Amplitudes in the cross section are scaled by maximum value.
4.6 Gaussian Width

The Gaussian filter (Equation 4.6), is applied to the data before the calculation of the receiver function and is a very gentle low pass filter with no phase distortion and a simple shape (Langston, 1979; Ammon, 1997). It has one parameter, the width $a$, that controls the frequency response. The choice made for this filter effects the receiver functions and so can have an effect on the results of the $H\kappa$ stacking. In general, a smaller Gaussian removes the higher frequencies, and results in mostly longer periods in the receiver function. These longer periods are less affected by gradient boundaries as they tend to see them as steps in velocity, but there is a corresponding loss of resolution, particularly in $V_p/V_s$. The higher Gaussians tend to have much better resolution, but have less sensitivity to gradational boundaries and can have more trouble with noise.

$$G(w) = e^{-w^2/(4a^2)} \quad (4.6)$$

Prior studies have made different choices for the Gaussian width $a$, and it is important to recognize that the results can vary with this choice. Zhu and Kanamori (2000) used a Gaussian width of 5.0 while Gilbert and Sheehan (2004) used 2.0 and Li et al. (2002) used 1.0. Within EARS we have used 2.5, but have also made comparisons with a Gaussian width of 1.0.

The smaller Gaussian widths generate a much broader peak in the $H\kappa$ stack, and so subtle amplitude variations, perhaps due to noise, can more easily effect the location of the maximum. In cases where there are multiple peaks in the $H\kappa$ stack that are of similar amplitude, changing the Gaussian can change the relative amplitudes, causing a different global maximum. This maximum hopping can yield very different results depending on the Gaussian chosen.

One can see the effect of the Gaussian width on the $H\kappa$ stack quite clearly with
XA.MM01 (for location see Figure 4.16), comparing a Gaussian width of 2.5 Hz and 1.0 Hz. The global maximum occurs at a thickness of 35.4 km for a Gaussian of 2.5 (Figure 4.7), while the global maximum is 46.5 km for a Gaussian width of 1.0 (Figure 4.8). For width of 2.5, the global maximum is quite clearly in the mid thirties with only a hint of amplitude in the 45-50 km range near $1.8 \frac{V_p}{V_s}$. However with a Gaussian width of 1.0 Hz, the situation is completely reversed with the obvious maximum near 46 km thickness and the remnants of the maxima near 32 km, with a very small $\frac{V_p}{V_s}$. With a width of 0.7 Hz, the situation again reverses, although not as dramatically, with the global maximum at 31.5 km but still with significant amplitude within the 45-50 km range. The loss of resolution in $\frac{V_p}{V_s}$ can also be seen with values between 1.72 and 1.93 all being plausible in the 1.0 Hz case. Thus, the estimate of the thickness can depend greatly on the choice of Gaussian. Without external knowledge of the crustal structure near this station, it is difficult to prefer one solution over the another.
Figure 4.7: $H_\kappa$ stack for XA.MM01 for a Gaussian width of 2.5

Figure 4.8: $H_\kappa$ stack for XA.MM01 for a Gaussian width of 1.0
4.7 Error bounds

There are two predominant methods of determining errors in thickness and $V_p/V_s$ within the $H_\kappa$ stacking technique. The first estimates the errors based on the curvature of the peak, with a broad maximum having a larger error than a sharper maximum. The second, bootstrapping, makes use of resampling the population of receiver functions to a set of maxima on which traditional statistical measurements can be applied. Within the $H_\kappa$ stack, there are two primary types of errors in making the correspondence between the maximum value of the stack and a thickness-$V_p/V_s$ model of the crust. The first is the broadness of the peak, with the error reflecting the likelihood that noise or processing artifacts have shifted the maximum away from the true value. The second is the likelihood that another separate maximum corresponds to the thickness and $V_p/V_s$ of the crust, and just happens to be slightly smaller than the global maximum. This could be due to noise, bad data or other boundaries in the crust or mantle that generate stronger reverberations. The error method of estimating the curvature does a good job for quantifying the first type of error, but offers no information about the second. Bootstrapping seems to bring both types of errors into the estimate, as both broadness of the peak and the existence of multiple, similar maxima can contribute. Because even a relatively broad maximum is still usually well constrained in thickness, a large error from the bootstrap usually means that there are multiple maxima, and that the results probably need further review to determine which is the correct maxima. The existence of multiple maxima can be viewed as a problem in determining crustal thickness automatically, but can also be viewed as a source of information. Multiple maxima are an indication that there is something more complex than a single layered crust, and so those stations are deserving of further study.
4.7.1 Curvature error bounds

Zhu and Kanamori (2000) estimate errors by using the second derivative of the surface with respect to $H$ and $\kappa$, yielding the following:

$$\sigma^2_H = -2\sigma_s^2 / \frac{\partial^2 s}{\partial H^2}$$
$$\sigma^2_\kappa = -2\sigma_s^2 / \frac{\partial^2 s}{\partial \kappa^2}$$

where $\sigma_s^2$ is the variance of the individual $H\kappa$ plots evaluated at the global maximum.

Julià and Mejía (2004) noted that this assumes that the variance of $H$ and $\kappa$ are uncorrelated, which is unlikely given the known trade-offs between thickness and $V_p/V_s$ (Ammon et al., 1990). They expand this to include the cross-derivatives, yielding:

$$\sigma^2_H = -2\sigma_s^2 \frac{\partial^2 s}{\partial H \partial \kappa} \left( \frac{\partial^2 s}{\partial H^2} \frac{\partial^2 s}{\partial \kappa^2} - \left( \frac{\partial^2 s}{\partial H \partial \kappa} \right)^2 \right)$$
$$\sigma^2_\kappa = -2\sigma_s^2 \frac{\partial^2 s}{\partial H^2} \left( \frac{\partial^2 s}{\partial H^2} \frac{\partial^2 s}{\partial \kappa^2} - \left( \frac{\partial^2 s}{\partial H \partial \kappa} \right)^2 \right)$$
$$\sigma^2_{H\kappa} = -2\sigma_s^2 \frac{\partial^2 s}{\partial H \partial \kappa} \left( \frac{\partial^2 s}{\partial H \partial \kappa} \left( \frac{\partial^2 s}{\partial H^2} \frac{\partial^2 s}{\partial \kappa^2} - \left( \frac{\partial^2 s}{\partial H \partial \kappa} \right)^2 \right) \right)$$

Setting the cross derivatives to zero in Equation 4.8 recovers Equation 4.7. Both of these expressions make the assumption that the maximum can be approximated by a bivariate Gaussian. Simply applying a standard finite difference approximation for the partials suffers from errors introduced by small variations from a smooth surface at neighboring values, and hence fitting by a known smooth surface is preferable. They also both suffer from the inability of multiple maxima to influence the error estimate.
4.7.2 Bootstrap error bounds

Bootstrapping is a technique for calculating errors on the maxima within a stack (Efron and Tibshirani, 1991). The procedure is to perform a sampling with replacement from the pool of receiver functions to create a new set of the same size. The value of the global maxima from the stack on this new set is noted, and the procedure is repeated a number of times. At the end, instead of the $H$ and $\kappa$ corresponding to a single maxima, there is a population of $(H, \kappa)$ pairs that can be subjected to normal statistical analysis of variance and standard deviation.

If the maxima is nearly the same regardless of the resampling of the receiver functions, then the result is robust in that it does not depend strongly on the underlying earthquakes. In this case the variance of the bootstrap will be small. If the maxima from the resamplings vary significantly, then the peak corresponding to the maximum will not be sharp and the variance of bootstrap will be large. Multiple maxima also contribute to large errors in the bootstrap. If there are several maxima whose amplitudes are reasonably close, then within each bootstrap iteration there is a reasonable probability that one besides the global maxima will be the maximum. If this happens frequently in the bootstrap, then the variance will be large, giving a second indication that the result is not to be trusted without further review.

Figure 4.9 shows a sample of $H\kappa$ stacks from the bootstrap of station TA.S08C, see Figure 4.4 for the station location. The original $H\kappa$ stack is shown at the top. The global maximum overall is at a thickness of 38 kilometers and a $V_p/V_s$ of 1.76. The next largest local maximum occurs at a thickness of 31 kilometers and a $V_p/V_s$ of 2.0. Each of these maxima are the largest within some bootstrap stacks, yielding a larger error than would be expected from just the broadness of the global maximum. The distribution of maxima for 100 bootstrap iterations can be seen in Figure 4.10. There is a majority clustering around the global maximum at 38 kilometers and $V_p/V_s$ of 1.76, labeled A, but also a number that cluster around the smaller local maximum at
31 kilometers and $V_p/V_s$ of 2.01, labeled B.
Figure 4.9: Various bootstrap $H\kappa$ stacks for TA.S08C. The original $H\kappa$ stack is at the top. The EARS estimate and the next largest local maximum are shown as open green and blue circles respectively. Each bootstrap is formed by stacking events chosen with replacement from the original successful list of events. Note that although all $H\kappa$ stacks show two local maxima, near 31 kilometers (blue) and 38 kilometers (green), which one is the largest in a particular bootstrap iteration, shown as a white dot, depends on which set of events are in the bootstrap. This results in a larger error for this station, and probably indicates the need for analyst review.
Figure 4.10: Global maxima for each of 100 bootstrap iterations for TA.S08C. Most cluster around the global maximum of 38 km thickness and $V_p/V_s$ of 1.76 (A) but a small number cluster around a local maximum at 31 km thickness and $V_p/V_s$ of 2.0 (B). The standard deviations for both thickness and $V_p/V_s$ from the bootstrap are shown by the error bars, ±0.067 in $V_p/V_s$ and ±2.4 kilometers in thickness.
4.8 HKStack Complexity

A problem with the $H\kappa$ stacking method is the fact that multiple maxima may occur and the largest amplitude maxima is not always the correct choice. This can be due to multiple real discontinuities in the earth as well as the coincidental alignment of noise, although the latter is less likely as the number of independent events increases. BK.CMB, shown in Figure 4.11, illustrates this with three large local maxima, see Figure 4.4 for station location. The maxima at 37 km most likely corresponds to the crust, even though it is slightly smaller in amplitude than the global maximum at 56 km. In the absence of outside knowledge of typical crust in the area it is difficult to automatically determine when a smaller local maximum is more appropriate than the global maximum, and in general this judgment needs to be made by an analyst. However, it is useful to have some quantitative measures of the “believability” of the result and if the global maxima is a good representation of the true crustal structure.

One measure of the goodness of the maximum of the $H\kappa$ stack is how much of the power in the stack is due to the global maxima. If one maxima far exceeds any other local maxima, then in the absence of limited quantities of data or poor quality receiver functions, one is forced to either accept it as representing the crust or argue that the Moho does not produce a strong signal for this station. We introduce the complexity of the $H\kappa$ stack as the ratio of the power in the residual $H\kappa$ plot to the power in the original stack. The residual is the difference between the original stack and an $H\kappa$ stack based on synthetic receiver functions for a crustal model that corresponds to the global maxima appropriately scaled to match the peak amplitude (Figure 4.12). This effectively subtracts the amplitude that would be expected from a discontinuity at this depth, both allowing easier identification of smaller local maxima and the calculation of the residual amplitude in the stack. An $H\kappa$ stack by its nature is very insensitive to ray parameter, and hence epicentral distance, and so there is no need to compute the synthetics at distances corresponding to all the earthquakes.
Figure 4.11: $H\kappa$ stack for BK.CMB showing three local maxima. The global maximum is the deepest of the three, at 56 kilometers thickness, while the shallower maximum at 37 kilometers likely corresponds to the true crustal thickness. The presence of the three local maxima contributes to a large complexity of .86. See Figure 4.4 for the station location.

used in the stack, nor even a range of distances.

The complexity for the $H\kappa$ stack for BK.CMB is 0.86, Figure 4.11, which is quite large, and is due to the presence of three similar amplitude maxima. In contrast, the $H\kappa$ stack shown in Figure 4.13 for TA.S08C is 0.59, reflecting the fact that the global maximum is significantly larger in amplitude than the secondary maximum. However, the second maximum still exists and the global maximum is broader than is predicted by the synthetics, yielding a moderately large complexity. Station TA.J06A, Figure 4.14, is an example of a much simpler $H\kappa$ stack. In this case there is a single well-defined maximum and very little else yielding a small complexity value of 0.33.

While a large complexity is certainly an indicator of unreliable results, without some comparison it is hard to interpret what values are “good” and what values are “bad”. We have chosen to make a comparison with the Crust2.0 (Bassin et al., 2000)
Figure 4.12: Synthetic (top) and Residual (bottom) $H\kappa$ stacks for BK.CMB. The synthetic is created for the local maximum at 56 kilometers thickness and the residual $H\kappa$ stack on the bottom shows the amplitudes that remain after the subtraction of the synthetic from the original shown in Figure 4.11.
Figure 4.13: $H\kappa$ stack for TA.S08C showing a global maximum at 38 kilometers thickness as well as a smaller secondary local maximum at 31 kilometers yielding a moderate complexity of .59. See Figure 4.4 for the station location.

Figure 4.14: $H\kappa$ stack for TA.J06A showing a well defined single maximum at 36 kilometers thickness yielding a low complexity of .33. See Figure 4.4 for the station location.
global crustal model. Crust2.0 divides the world into \(2^\circ \times 2^\circ\) cells and assigns to each cell one of 360 profiles. The profiles and their assignment to cells is based on refraction and reflection seismic studies as well as receiver function studies in areas where this data exists, and by using average values for areas of like tectonic setting and crustal age where it does not (Laske et al., 2007). Because the Crust2.0 model is based on \(2^\circ \times 2^\circ\) cells and EARS is station-based, there should be variation and small disagreements, but it provides a globally uniform baseline to make comparisons.

Figure 4.15: Cumulative histogram of complexity versus the difference between Crust2.0 (Bassin et al., 2000) and EARS. Low complexity values, less than .5 and shown in red, are clearly in better agreement with the Crust2.0 model on average than are stations with high complexity, greater than 0.7 and shown in blue.
Figure 4.15 shows a cumulative percentage histogram of number of stations versus the difference with Crust2.0 for three ranges of complexity values. These three ranges, below 0.5, from 0.5 to 0.7 and above 0.7 were chosen as they roughly divide the EARS results into thirds, with 436, 557 and 481 stations respectively. The mean for the ranges is 4.3, 6.8 and 11.3, with standard deviations of 4.1, 6.8 and 9.8 respectively, clearly indicating that higher complexity and difference with Crust2.0 are related. For low complexity stations (less than 0.5) 73% are within 5 kilometers of Crust2.0. For a medium complexity (0.5 to 0.7) this drops to 56%, and for the highest complexity (over 0.7) this falls to 34%.
4.9 Comparison and Calibrations

Because EARS is a highly automated system, and therefore there is less human oversight, it is very useful to make a comparison between the results from prior studies and the results for the same stations or same area in EARS. We have chosen two studies for this. The first is Li et al. (2002), a study of crust and upper mantle structure for stations in the 1995 Missouri to Massachusetts Transect experiment using receiver functions, although not the $H\kappa$ method. Being a Program for the Array Seismic Studies of the Continental Lithosphere (PASSCAL) deployment, it provides a good comparison for results from other temporary stations. The second is Zhu and Kanamori (2000) which introduced the $H\kappa$ stacking technique, and so provides a useful comparison without variations due to differing methods. In addition, the study area of southern California contains a large number of permanent stations and is the initial deployment area for USArray.

4.9.1 MOMA

The 1995 Missouri to Massachusetts Transect PASSCAL experiment provides a good opportunity for comparison with EARS because the duration of the deployment is similar to the planned USArray Transportable Array stations. The MOMA experiment deployed 18 stations forming a line between IU.CCM, Cathedral Cave, Missouri, and IU.HRV, Harvard, Massachusetts, see Figure 4.16. Li et al. (2002) estimated crust and upper-mantle structure using receiver functions. They used a suite of three assumed crustal P velocities and Vp/Vs ratios: 6.6 km/s with 1.84, 6.6 km/s with 1.80, and 6.5 km/s with 1.73, generating up to three crustal thickness estimates for each station using a Gaussian width of 1.0. They did not generate crustal thickness estimates for two stations, MM11 and MM15. Figure 4.17 shows a comparison of the crustal thickness estimates from their receiver function analysis with those of EARS.
Overall there is good agreement, with stations in the western end showing the largest differences.

Figure 4.16: Stations locations within the MOMA array used in Li et al. (2002).

The eastern stations, with the exception of MM01, generally fall within or near the estimates of Li et al. (2002). MM04 through MM07 are in a region that has a significantly different Vp in the Crust2.0 model from those used in the MOMA study, almost .5 km/s slower, 6.1 km/s versus 6.6 km/s. Because the crustal thickness varies close to linearly with Vp, with a .1 km/s increase in Vp causing almost a .5 km increase in crustal thickness, it is easy to attribute up to 2.5 km of the difference for these stations just to the assumed Vp. EARS shows MM01 with a crust 10km thinner than the closest estimate and with relatively large standard deviation. As shown in Figure 4.18, there is a single global maximum at 34km and no local maxima that might correspond to a 45 km crust at a Gaussian width of 2.5. As discussed in section 4.6, the differing Gaussians between EARS (2.5) and Li et al. (2002) (1.0) can explain much of this large difference. Repeating the EARS processing but with a Gaussian of 1.0 results in a 46 km crust, see Figure 4.8, in much better agreement with
Figure 4.17: Crustal thickness comparison with Li et al. (2002). Red bars show the range of values from Li et al. (2002), which had up to three values per station as they used three fixed values of $V_p$ and $V_p/V_s$ (% 6.6 km/s with 1.84, 6.6 km/s with 1.80, and 6.5 km/s with 1.73). Li et al. (2002) did not estimate a thickness at MM11 and MM15. The EARS result for MM17 is at 60 km, off the bottom of the figure. The black circles represent the EARS results, computed with a Gaussian of 2.5. The green circles were generated by the same EARS processing system, but with a Gaussian of 1.0, illustrating the differences that can occur due to the Gaussian filter. The error bars are from the bootstrap for Gaussian of 2.5.
Li et al. (2002). However, deciding between the two values requires some external knowledge of the local crustal structure that is not available within the automated processing system. The EARS value of 34 is also midway between the two adjoining stations of IU.HRV and MM02, which both agree well.

The western stations are more of a problem, with MM13, MM16, MM17, and MM18 showing large standard deviations in EARS and significant differences with Li et al. (2002). Figure 4.18 shows the $H\kappa$ stacks for these stations. Of these, MM16 has the smallest difference, 5km deeper, and has the most compelling thickness versus Vp/Vs maxima. While the error bars are large at this station, due to local maxima influencing the bootstrap, it is consistent with neighboring stations MM14 and MM17 in Li et al. (2002) and the EARS result at MM15. MM13 and MM18 are more of a problem, as both are shallower in EARS, 6km and 9km respectively, and significantly different from neighboring stations. MM13 has a local maxima at 43km that would be more consistent with both Li et al. (2002) as well as a more reasonable Vp/Vs ratio, and is hence likely a case of the correct maxima not being the global maxima in the stack. MM18 is more variable but does not have any maxima that might correspond to a 44-47 km crust. In addition, MM18 and MO18 are treated as two separate stations in EARS, but are combined in Li et al. (2002). Both MM13 and MM18 have moderately high complexity, 0.62 and 0.66 respectively, indicating they are less reliable results. MM17 is off scale, with a thickness estimate of 60 kilometers in EARS. It only has a single large maximum, and hence no local maxima that might be preferred. Due to the large standard deviations at these four western stations, along with MM01, they are probably indicators of problems in the automated system and may require an analyst to make a quality judgment. As with all temporary seismic networks, the number of earthquakes is small relative to permanent stations. This may explain why the two permanent stations at the ends of the array have very good agreement in both studies, as well as very small standard deviations in EARS.
Figure 4.18: $H\kappa$ stacks for XA stations MM01, MM11, MM13, MM16, MM17, MM18 for a Gaussian of 2.5. The global maximum for each station is marked with a white dot.
4.9.2 Zhu and Kanamori

The \( H_k \) stacking technique was first applied by Zhu and Kanamori (2000) to stations in southern California (Figure 4.19). The 70 thickness results they produced are shown in Figure 4.20 as a surface, created by averaging over 0.25 × 0.25 degree bins fitting a minimum curvature surface via the GMT `surface` command (Wessel and Smith, 1991). Empty bins that are more than 0.75 degrees from a bin with data are white. A corresponding surface using EARS results is shown in Figure 4.21. Overall there is very good agreement, with only minor differences in the overall pattern of crustal thickness, with perhaps the biggest difference being that EARS has access to more stations and is able to generate results further to the northwest into central California as well as further to the northeast in the basin and range of Nevada. These additional stations show the thickened crust of the Sierra Nevada Mountains. Thin crust is found near the coast and in the Salton Sea of southernmost California.
Figure 4.19: Topography of southern California, with the stations used in the EARS results in Figure 4.21 shown as blue triangles, stations used in the Zhu and Kanamori (2000) results in Figure 4.20 in red. Stations used in both are shown in magenta. The 23 stations in Zhu and Kanamori (2000) that were not used in the EARS results are not available from the IRIS DMC.
Figure 4.20: Crustal thickness from the 70 stations in Zhu and Kanamori (2000) shown as a minimum curvature surface after binning the stations in 0.25 × 0.25 degree bins. The stations used are shown as green and red triangles.
Figure 4.21: Crustal thickness from EARS using the 165 stations shown in Figure 4.19 shown as a minimum curvature surface after binning the stations in $0.25 \times 0.25$ degree bins. The stations used are shown as blue and green triangles.
4.10 Western US

The USArray component of EarthScope is installing a temporary broadband seismic network composed of 400 three-component broadband seismometers at a spacing of about 70 kilometers that will roll across the United States over the course of 12 years. EARS was created with the notion of the calculation of bulk crustal properties following USArray as it moves across the continent, with new stations being included as they are deployed and each station’s result being updated as new data arrives. Because the USArray deployment starts on the west coast and moves east, only the west coast has sufficient data at this point to generate the dense results that USArray should bring to the entire country.

The crustal thickness from EARS shows variation from 20 kilometers along the coast to over 40 kilometers in western Washington and Oregon and the Sierras of California, see Figure 4.22. Thinner values are seen along the coast and in southern California with thicker values under the Sierra Nevada mountains and inland from the Washington and Oregon coasts. The surface is created from all stations with at least five events and where the complexity is less than 0.7. The stations thicknesses are averaged over $0.25 \times 0.25$ degree bins and a minimum curvature surface is fit via the GMT surface command (Wessel and Smith, 1991). Empty bins that are more than 0.75 degrees from a bin with data are white. This view of the EARS data allows broad patterns of thickness to be recognized.

Perhaps the most striking feature in Figure 4.22 is the north-south trending band of thick “crust” in western Washington and Oregon. Bostock et al. (2002) have suggested that seepentinization of the upper mantle above the subducting Juan de Fuca plate may obscure the signal from the base of the continental crust. This, coupled with the ability of the Moho of the subducting oceanic slab to generate a large signal (Owens et al., 1988), lead us to interpret this east-dipping feature as the subducting Juan de Fuca plate. The thickness estimates from EARS agree well with
Figure 4.22: Unreviewed crustal thickness estimates for the western US based on averages within $0.25 \times 0.25$ degree bins. Empty bins that are more than 0.75 degrees from a bin with data are white.
the slab model of Flück et al. (1997), showing the top of the slab descending from about 20 kilometers along the coast to 50 kilometers near -123 longitude in Oregon and Washington.

Figure 4.23: Cross section of the descending Juan de Fuca plate, modified from the results of Bostock et al. (2002), showing the thickness estimates from EARS from 42° to 45.5° latitude, with no vertical exaggeration. Stations to the west of 122.3° longitude correlate well with the subducting oceanic crust while stations to the east correlate well with the continental Moho.

Figure 4.23 shows the descending Juan de Fuca plate and all EARS results between 42° to 45.5° latitude with at least 5 earthquakes and a complexity less than 0.7 overlain. To the east, the thickness estimates agree well with a 36 kilometer thick continental crust. Estimates to the west correlate well with the subducting oceanic crust. This correlation continues into the area of serpentinized mantle in the forearc. The station near the eastern edge of the forearc, TA.G04A, is more ambiguous, but is still closer to the depth of the slab than to the continental Moho. Our technique does not yet take dipping structures into account, and so the slope of the slab is a source of error. Bostock et al. (2002) have also suggested that the level of sepentinization may be high enough to invert the Moho, resulting in lower S velocities in the mantle than in the
overlying crust. It may be possible to identify this within the \( H\kappa \) stacking technique by looking for a minimum instead of maximum as the amplitudes of all three phases should be inverted.

The details of these thickness maps need to be interpreted with care due to the many possible sources of error. At each station the thickness value represents the maximum amplitude in the \( H\kappa \) stack. This value ideally should correspond to the largest change in material properties, but this may not be true in the presence of noise. In particular, stations with small numbers of events can often have unreliable results. Also, the largest change in material properties may not correspond to the base of the crust. In particular, the boundary observed in western Washington and Oregon may correspond to the subducting slab \((\text{Bostock et al.}, 2002; \text{Broucher et al.}, 2003)\) instead of the base of the continental crust. Lastly, the surface represents an interpolation amongst the stations, and hence is subject to lack of detail in areas without sufficient coverage. Moreover, one outlying station can have an effect on the surrounding areas.

It is useful to return to the individual station results to test the validity of any feature in the surface. Figure 4.24 shows the stations that were used in the generation of the surface in figure 4.22, aiding in the identification of outlying data points. For example, the estimated 40 kilometer thick crust in northwestern Nevada in Figure 4.22 is unlikely to be real, and looking at the individual station results reveals that this large value is due to a single station, TA.M08A, whose \( H\kappa \) stack has a maximum at 47 kilometers. There are eight neighboring stations, shown in Table 4.1, all of which have thickness estimates between 29 and 37 kilometers, and so it seems likely that this station’s result is unlikely to correspond to the true crustal thickness.
Figure 4.24: Stations in EARS with thickness estimates within the western US. All stations that contributed to the surface in Figure 4.22 are shown.
<table>
<thead>
<tr>
<th>Station</th>
<th>Lat/Lon</th>
<th>Thickness (km)</th>
<th>$V_p/V_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA.M08A</td>
<td>41.45/-118.38</td>
<td>47 km</td>
<td>1.69</td>
</tr>
<tr>
<td>TA.L08A</td>
<td>42.19/-118.34</td>
<td>32 km</td>
<td>1.84</td>
</tr>
<tr>
<td>TA.N08A</td>
<td>40.78/-118.13</td>
<td>31 km</td>
<td>1.83</td>
</tr>
<tr>
<td>TA.L09A</td>
<td>42.02/-117.67</td>
<td>37 km</td>
<td>1.86</td>
</tr>
<tr>
<td>TA.N07B</td>
<td>40.78/-118.97</td>
<td>29 km</td>
<td>1.81</td>
</tr>
<tr>
<td>TA.L07A</td>
<td>42.02/-119.34</td>
<td>35 km</td>
<td>1.83</td>
</tr>
<tr>
<td>TA.M09A</td>
<td>41.42/-117.45</td>
<td>35 km</td>
<td>1.60</td>
</tr>
<tr>
<td>TA.M07A</td>
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<td>34 km</td>
<td>1.76</td>
</tr>
<tr>
<td>TA.N09A</td>
<td>40.85/-117.52</td>
<td>32 km</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Table 4.1: EARS results for stations near TA.M08A
Bibliography


Ahern, T., What happened to FISSURES?-or-Exactly what is the data handling interface?, *IRIS DMC Newsletter*, 3(3), 3, 2001b.


